Term Test #2
ACT 2120 – Interest Theory
Wednesday Feb. 28th, 4:00 PM - 5:15 PM
Instructor: Jeff Strong

There are 10 Questions worth 5 marks each.
50 marks total.
Show all your work.

1.) The present value of a series of 50 payments starting at 100 at the end of the first year and increasing by 1 each year thereafter is equal to X. The annual effective rate of interest is 9%.

Calculate X.

2.) Dean makes a series of payments at the beginning of each year for 20 years. The first payment is 100. Each subsequent payment through the tenth year increases by 5% from the previous payment. After the tenth payment, each payment decreases by 5% from the previous payment.

Calculate the present value of these payments at the time the first payment is made using an annual effective rate of 7%.

3.) At a nominal interest rate of j convertible semi-annually, an investment of 1000 immediately and 1500 at the end of the first year will accumulate to 2600 at the end of the second year.

Calculate j.

4.) You have just won a lottery that pays \$1,000 per month in the 1st year, \$1,100 per month in the 2nd year, \$1,200 per month in the 3rd, and so on. Payments are made at the end of each month for 10 years.

Using an effective annual interest rate of 3%, calculate the present value of this prize.

5.) You are given two series of payments. Series A is a perpetuity with payments of 1 at the end of each of the first 2 years, 2 at the end of each of the next 2 years, 3 at the end of each of the next 2 years, and so on. Series B is a perpetuity with payments of K at the end of each of the first 3 years, 2K at the end of each of the next 3 years, 3K at the end of each of the next 3 years, and so on. At an effective interest rate of 7.2% the present values of the two series of payments are equal. Calculate K.

6.) Plastic trays last 8 years and cost 20. Metal trays last 24 years and cost x. Trays are needed for 48 years, and inflation will increase the cost of the trays 5% year. At 10.25% interest, determine x so that the buyer is indifferent to purchasing plastic or metal trays.

7.) University tuition currently costs $5000 per year and is paid at the beginning of the school-year. Your newborn child will enter university exactly 18 years from today. You would like to save for your child's tuition assuming she will go to university for 7 years. The cost of tuition is expected to increase by $150 per year over the next 10 years. After ten years tuition will increase at an effective rate of 25% per 4 year period, forever. You plan to save for tuition by making level monthly deposits at the end of each month for 18 year starting today. Determine your monthly deposit assuming that your account earns a nominal rate of 12% compounded monthly.

8.) Jane receives a 10-year increasing annuity-immediate paying 100 the first year and increasing by 100 each year thereafter.

Mary receives a 10-year decreasing annuity-immediate paying x the first year and decreasing by x/10 each year thereafter.

At an effective annual interest rate of 5%, both annuities have the same present value.

Calculate x.

9.) You are the purchasing manager for your company's warehouse. All of the light-bulbs in your warehouse recently burnt out. Available for purchase are two different types of bulbs: Type A that last for 5 years and produce 6 Kilowatts of power, and Type B that last for 2 years and produce 2 Kilowatts of power. The price of Type A bulbs is decreasing by 1% each year while the price of type B bulbs decreases by 4% each year.

You are also given the following:

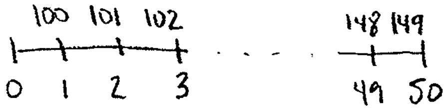
- You would need to keep your warehouse lit for the next 20 years.
- It takes 12000 Kilowatts of power to light your warehouse.
- The annual effective interest rate is 5.5%.
- Currently Type A bulbs cost 5 times as much as Type B.
- Type B bulbs cost $1.

Assuming that both type A and type B bulbs will be available for sale over the next 20 years how much will you save buying Type A bulbs instead of Type B in today's dollars?

10.) Investment X for 100,000 is invested at a nominal rate of interest, j, convertible semi-annually. After four years it accumulates to 214,358.88. Investment Y for 100,000 is invested at a nominal rate of discount, k, convertible quarterly. After two years it accumulates to 232,305.73. Investment Z for 100,000 is invested at an annual effective rate of interest equal to j in year one and an annual effective rate of discount equal to k in year two. Calculate the value of investment Z at the end of two years.

Term Test #2 Solutions.

(1.)



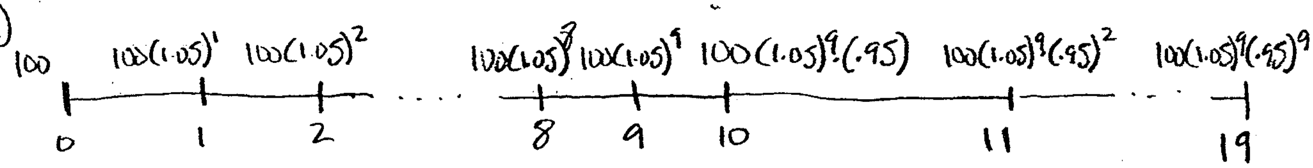
$P=100,\ Q=1$

$$PV = 100 \cdot a_{\overline{50}|.09} + 1 \cdot \left[\frac{a_{\overline{50}|.09} - 50v^{50}}{.09} \right] = X$$

$$= 100(10.9617) + 1 \cdot \left[\frac{10.9617 - .6724}{.09} \right] = 1096.17 + 114.325$$

$$= 1210.50$$

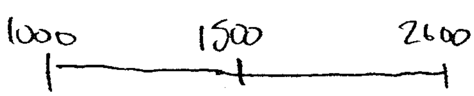
(2.)



$$PV = 100\left[\frac{1-\left(\frac{1.05}{1.07}\right)^{10}}{.07-.05}\right](1.07) + 100\cdot(1.05)^9(.95)\left[\frac{1-\left(\frac{.95}{1.07}\right)^{10}}{.07-{-.05}}\right]\cdot\frac{1}{1.07^9}$$

$$PV = 100[9.1995] + 100\cdot(1.4738)[5.7969]\cdot\frac{1}{1.8385} = 1384.65$$

(3.)



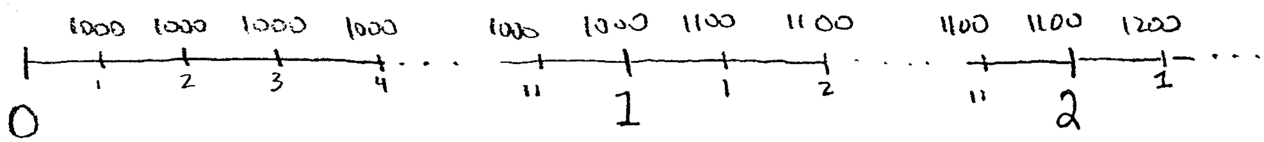
$j = i^{(2)}$ $\qquad 1000(1+i)^2 + 1500(1+i) = 2600$ $\qquad$ let $x = 1+i$

$\Rightarrow 1000x^2 + 1500x - 2600 = 0 \quad \Rightarrow x = 1.028342 \quad \Rightarrow i = 2.8342\%$

$$i^{(2)} = \left[(1+i)^{1/2} - 1\right] \times 2 = \left[(1.028342)^{1/2} - 1\right] \times 2 = .02814 \text{ or } 2.814\%$$

Term Test #2 Solutions.

(4)



$(1+i) = \left(1+\frac{i^{(12)}}{12}\right)^{12}$ $\qquad \frac{i^{(12)}}{12} = (1.03)^{1/12} - 1 = .24663\%$

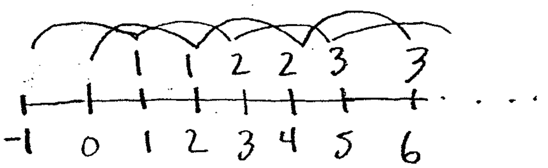
$P = 1000\, a_{\overline{12}|.24663} = 11809.82 \qquad Q = 100\, a_{\overline{12}|.24663} = 1180.982$

$$PV = \left[11809.82 \cdot a_{\overline{10}|.03} + 1180.982 \left[\frac{a_{\overline{10}|.03} - 10v^{10}}{.03}\right]\right] \cdot (1.03)$$

$$PV = \left(11809.82\,(8.5302) + 1180.982\left[\frac{8.5302 - 7.44094}{.03}\right]\right) \cdot (1.03)$$

$$PV = 103762.41 + 44166.30 = \underline{147928}$$

(5) Series A



This is 2 increasing perpetuities with payments every 2 years

$\frac{i^{(1/2)}}{1/2} = (1+i)^2 - 1 = (1.072)^2 - 1 = .149184$ $\qquad PV = \left[\frac{1}{.149184^1} + \frac{1}{.149184^2}\right] \times (1.072) +$

$$\left[\frac{1}{.149184^1} + \frac{1}{.149184^2}\right] = 51.6351 \cdot (1.072) + 51.6351 = 106.9879$$

ries B

let $P = k\, a_{\overline{3}|.072} = Q = k \cdot a_{\overline{3}|.072} = k \cdot 2.61476$

$' = (1.072)^3 - 1 = .231925 \Rightarrow PV = k\left[\frac{2.61476}{.231925^1} + \frac{2.61476}{.231925^2}\right] \cdot (1.231925)$

$\cdot 73.77408 \Rightarrow 106.9879 = k \cdot 73.77408 \Rightarrow \boxed{k = 1.45}$

Term Test #2 Solutions.

⑥ **Plastic Trays**

20	$20(1.05)^8$	$20(1.05)^{8^2}$	$20(1.05)^{8^3}$	$20(1.05)^{8^4}$	$20(1.05)^{8^5}$	
	8	16	24	32	40	48

let $(1+k') = 1.05^8$

$k' = .4775$ or 47.75%

$$\frac{i^{(1/8)}}{1/8} = (1.1025)^8 - 1 = 118.287\%$$

$$PV = 20\left[\frac{1-\left(\frac{1.4775}{2.1829}\right)^6}{1.18287-.4775}\right]\cdot(2.18287)^1$$

$$PV = 20\,[1.281306]\cdot(2.18287) = 55.94$$

Metal Trays.

x	x	
0	24	48

$$PV = x + \frac{x(1.05)^{24}}{1.1025^{24}} = 1.31006x \Rightarrow 55.94 = 1.31006x$$

$$x = \underline{\underline{42.70}}$$

⑦ PV of deposits $= X\cdot a_{\overline{12\cdot 18}|\frac{.12}{12}} = X\cdot 88.34309$

· Tuition cost 18 years from today $\Rightarrow (5000 + 150\cdot 10)\cdot(1.25)^2 = 10156.25$

· let $(1+k') = (1.25)^{1/4}$

$k' = 5.737126\%$

$$PV \text{ of tuition} = 10156.25\left[\frac{1-\left(\frac{1.0574}{1.12683}\right)^7}{.12683-.0574}\right]\cdot v^{17}_{12.683\%}$$

$$i = \left(1+\frac{i^{(12)}}{12}\right)^{12} - 1 = 1.01^{12} - 1 = .12683$$

$$= 10156.25\cdot[5.174449]\cdot(.131353)$$

$$= 6903$$

$\Rightarrow$ PV of deposits = PV of tuition

$X\cdot 88.34309 = 6903$

$X = \underline{\underline{\$78.14}}$

Term Test #2 Solutions.

(8) Jane: $PV = P \cdot a_{\overline{10}|} + Q\left[\dfrac{a_{\overline{10}|} - 10v^{10}}{i}\right] = 100(7.7217) + 100\left[\dfrac{7.7217 - 6.1391}{.05}\right]$

$PV = 3937.38$

Mary: $PV = P \cdot a_{\overline{10}|} - Q\left[\dfrac{a_{\overline{10}|} - 10v^{10}}{i}\right] = X \cdot (7.7217) - \dfrac{X}{10}\left[\dfrac{7.7217 - 6.1391}{.05}\right]$

$PV = 7.7217x - \dfrac{x}{10} \cdot [31.65205] = 4.5565x$

$\Rightarrow 3937.38 = 4.5565x \qquad x = \underline{\underline{864.12}}$

(9) Type A:

$\#\text{ of bulbs} = \dfrac{12000}{6} = 2000$

cost = \$5 PV of purchases $= 2000 \cdot 5\left[1 + \dfrac{(.99)^5}{(1.055)^5} + \dfrac{(.99)^{10}}{(1.055)^{10}} + \dfrac{.99^{15}}{1.055^{15}}\right]$

let $v = \left[\dfrac{.99}{1.055}\right]^5 \Rightarrow PV = 10000 \cdot \ddot{a}_{\overline{4}|(\frac{1}{v}-1)} = 10\,000\left[\dfrac{1 - \left(\frac{.99}{1.055}\right)^{5 \cdot 4}}{\left(\frac{1.055^5}{0.99^5} - 1\right)}\right] \cdot \left(\dfrac{1.055^5}{.99^5}\right)$

remember $i = \dfrac{1}{v} - 1$

$= 10000\,[1.922657](1.37432)$
$= 26423.37$

Type B

$\#\text{ of bulbs} = \dfrac{12000}{2} = 6000$

$PV = 6000 \cdot 1\left[1 + \dfrac{.96^2}{1.055^2} + \left(\dfrac{.96^2}{1.055^2}\right)^2 + \cdots \left(\dfrac{.96^2}{1.055^2}\right)^9\right] = 6000\, \ddot{a}_{\overline{10}|\frac{1.055^2}{.96^2}-1}$

$= 6000\, \ddot{a}_{\overline{10}|.2077\%} = 6000 \cdot 4.9336 = 29601.65 \Rightarrow \text{diff} = 29601.65 - 26423.37$
$= \underline{\underline{3178.29}}$

Term Test #2 Solutions.

(10.) Investment X:

$$100\,000\left(1+\frac{j}{2}\right)^{2\cdot 4} = 214\,358.88 \qquad j=\left(\left[\frac{214\,358.88}{100\,000}\right]^{1/8}-1\right)\times 2$$

$j = 20\%$

Investment Y:

$$100000\left(1-\frac{k}{4}\right)^{-4\cdot 2} = 232\,305.73 \qquad k = \left[1-\left(\frac{232\,305.73}{100\,000}\right)^{-1/8}\right]\times 4$$

$k = 40\%$

Investment Z:

$$100000(1+j)(1-k)^{-1} = 100000(1.2)(1-.40)^{-1} = \underline{\underline{200\,000}}$$